Interference & YDSE

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Wave equation, path difference and phase difference

Simulation
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Wave equation of a wave is

$$\boxed{y = A\cos(\omega t + \phi)}$$

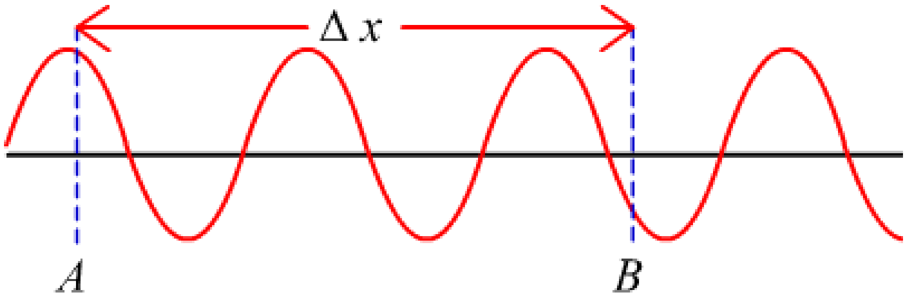
y : displacement A : amplitude ωt : time dependent phase
ϕ : initial phase or constant phase

Path difference (Δx) : Difference in lengths of optical paths of two waves or the optical path length between two points on a wave.

Phase difference ($\Delta \phi$) : Difference in the phases of two waves.

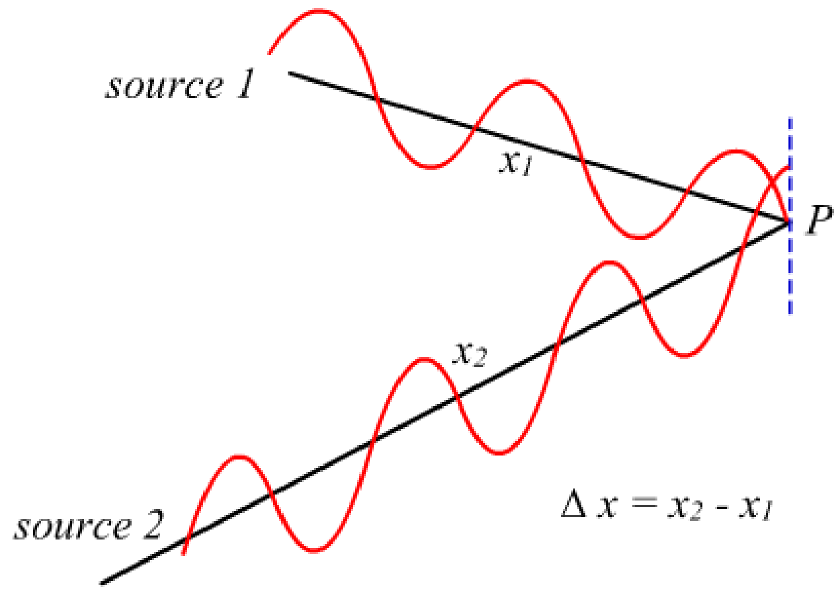
$$\boxed{\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x} \quad (1)$$

Relation between phase difference and path difference



Superposition (addition) of waves

Principle of superposition : If y_1 and y_2 are the displacements at the common point due to each wave then their resultant is given by



$$y_{\text{resultant}} = y_1 + y_2$$

If $y_1 = A \cos(\omega t)$ and $y_2 = A \cos(\omega t + \phi)$ then

$$y = 2A \cos\left(\frac{\phi}{2}\right) \cos\left(\omega t + \frac{\phi}{2}\right)$$

First cosine term represents the resultant amplitude.

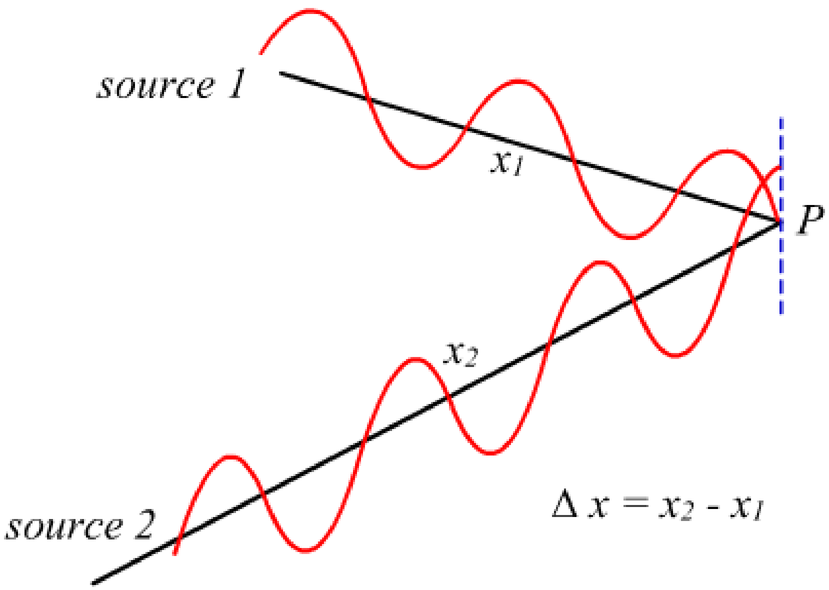
Intensity is proportional to square of amplitude therefore resultant intensity is given by

$$\boxed{I = 4I_o \cos^2\left(\frac{\phi}{2}\right)} \quad \text{—— } 2$$

Superposition (addition) of waves

Resultant intensity at any point on the screen is given by

$$\boxed{I = 4I_o \cos^2\left(\frac{\phi}{2}\right)}$$



In case of incoherent waves, phase difference (ϕ) is not constant. Therefore average of $\cos^2(\phi)$ function (i.e. ½) is used which results in a uniform illumination on the screen.

In case of coherent waves, phase difference between waves (ϕ) is constant. Therefore resultant intensity, at any point on the screen, is determined phase difference between superimposing waves arriving at that point.

Condition for maxima and minima

Resultant intensity at any point on the screen is given by

$$I = 4I_o \cos^2\left(\frac{\phi}{2}\right)$$

Condition for **maximum**
(constructive interference)

Phase difference $\phi = 2n\pi$

Path difference $\Delta x = n\lambda$

n = 0, 1, 2,3 …

Condition for **minimum**
(destructive interference)

Phase difference $\phi = (2n-1)\pi$

Path difference $\Delta x = (2n-1)\dfrac{\lambda}{2}$

n = 1, 2,3 …

A general approach : For any phenomenon related to superposition of waves, determine path difference based on geometry of the situation, obtain corresponding phase difference and use it in above equation to obtain the resultant intensity.

Resultant in a general case

Resultant amplitude due to superposition of two wave of amplitudes A_1 and A_2 having a phase difference ϕ between them is given by

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi)}$$

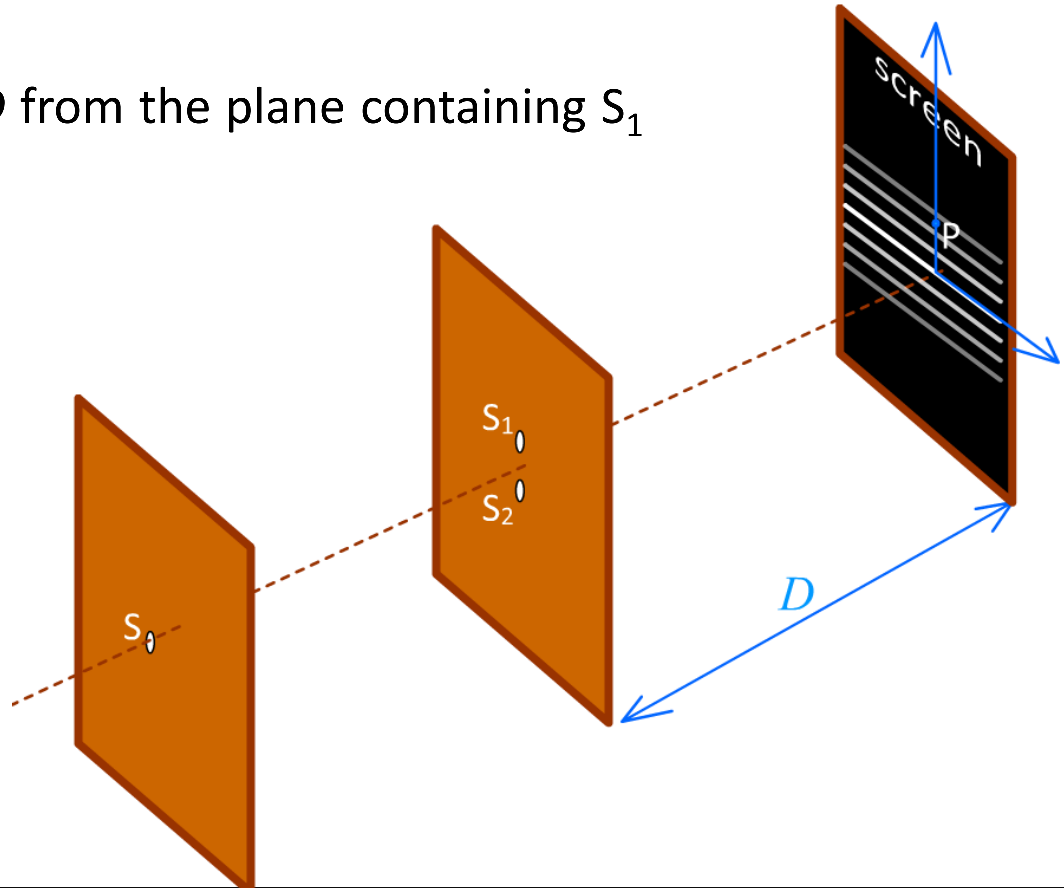
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi)$$

Note

- Amplitudes are added as vectors using a relation similar to parallelogram law of addition of vectors
- When two or more waves superimpose, the amplitudes are added and then, if required, intensity is obtained from the amplitude.

Young's double slit experiment (experimental setup)

- A pinhole is made in an opaque screen and light is allowed to pass through it
- Another opaque screen is placed parallel to the first screen with two pin holes S_1 and S_2 separated by a distance d and equidistant from S.
- This arrangement ensures a higher coherence in the waves emitted from S_1 and S_2.
- A screen is placed at a distance D from the plane containing S_1 and S_2. ($D >> d$)
- Light wave emerging from S_1 and S_2 superimpose each as they reach the screen
- Resultant intensity at any point on the screen is determined by the phase difference between the interfering waves as they reach the point

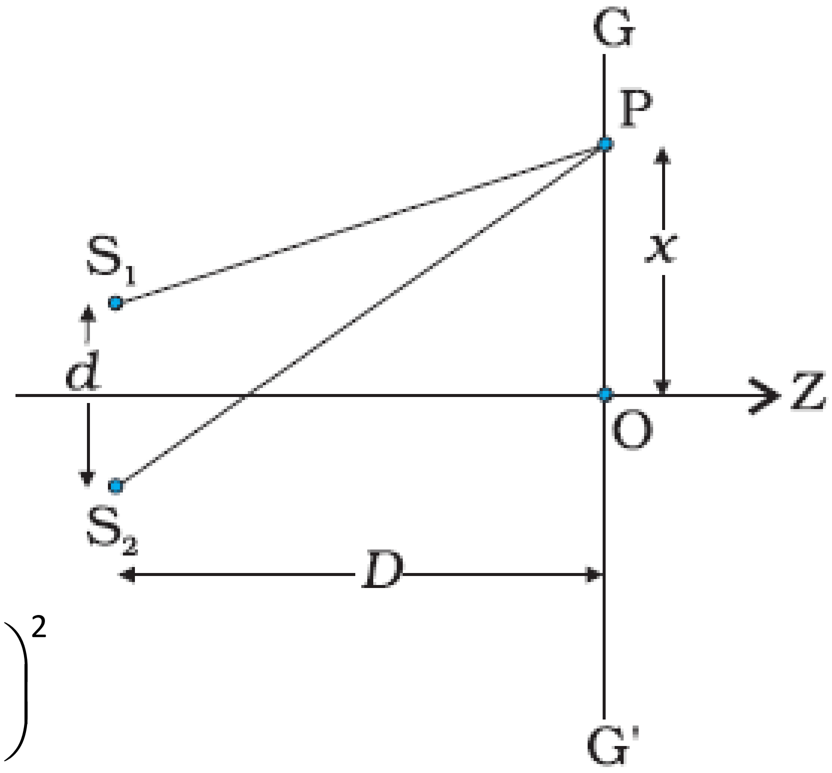


Y D S E (calculation of path difference)

P is a point at a distance x from the midpoint O on the screen. D is the distance of the screen from that plane of the slits S_1 and S_2. Distance between the slits is d. In the experimental setup $d << D$.

Path length of wave emanating from S_1 and reaching P is S_1P and path length of wave emanating from S_2 and reaching P is S_2P.

Path difference between the waves is $S_2P - S_1P$.



From the figure we get

$$S_1P^2 = D^2 + \left(x - \frac{d}{2}\right)^2$$

$$S_2P^2 = D^2 + \left(x + \frac{d}{2}\right)^2$$

$$S_2P^2 - S_1P^2 = D^2 + \left(x + \frac{d}{2}\right)^2 - D^2 - \left(x - \frac{d}{2}\right)^2$$

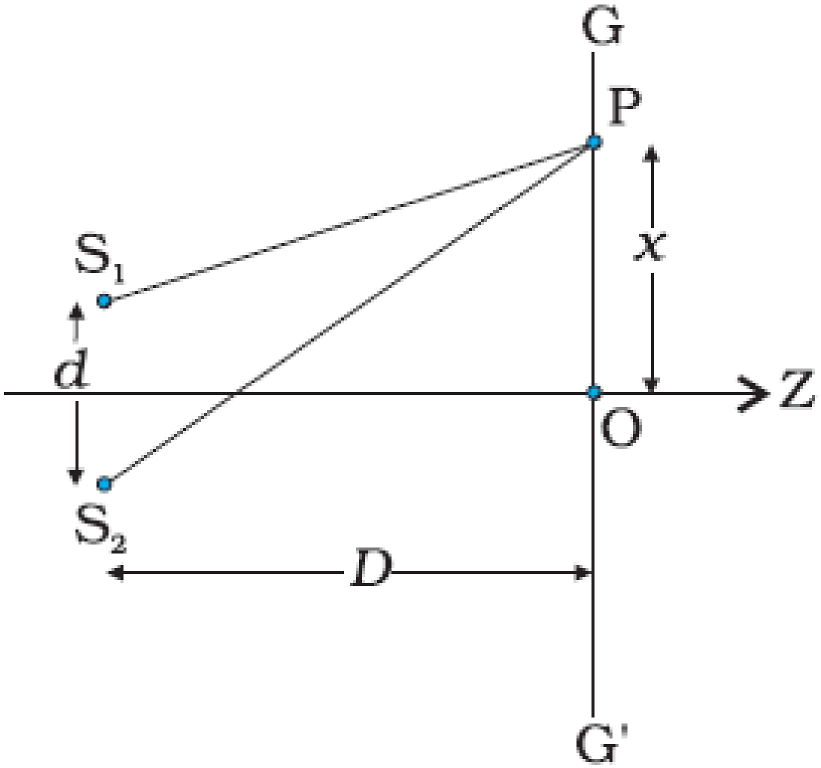
Y D S E (calculation of path difference)

$$S_2P^2 - S_1P^2 = D^2 + \left(x + \frac{d}{2}\right)^2 - D^2 - \left(x - \frac{d}{2}\right)^2$$

$$S_2P^2 - S_1P^2 = 2xd$$

$$\left(S_2P + S_1P\right)\left(S_2P - S_1P\right) = 2xd$$

In the experimental setup $D >> d$ therefore using the corresponding approximation we get



$$2D\left(S_2P - S_1P\right) = 2xd$$

$$\left(S_2P - S_1P\right) = \frac{xd}{D}$$

$$\boxed{\Delta x = \frac{xd}{D}} \quad 4$$

Y D S E (formation and location of fringes)

Comparing the relation for path difference in YDSE with the general conditions for constructive and destructive interference, the location of bright and dark bands (or fringes) can be obtained.

Relation for path difference for YDSE setup is $\Delta x = \frac{xd}{D}$

Relation for path difference for constructive interference is

$$\Delta x = n\lambda$$

Equating the relations we get

$$\frac{xd}{D} = n\lambda$$

$$\boxed{x_{\text{bright}} = n\left(\frac{\lambda D}{d}\right)}$$

Relation for path difference for destructive interference is

$$\Delta x = (2n-1)\frac{\lambda}{2}$$

Equating the relations we get

$$\frac{xd}{D} = (2n-1)\frac{\lambda}{2}$$

$$\boxed{x_{\text{dark}} = \frac{(2n-1)}{2}\left(\frac{\lambda D}{d}\right)}$$

Y D S E (bandwidth)

Band width (β) is defined as the distance between any two adjacent bright or dark bands)

Location of n^{th} bright band is given by

$$x_n = n\left(\frac{\lambda D}{d}\right)$$

Location of (n + 1) th bright band is given by

$$x_{n+1} = (n+1)\left(\frac{\lambda D}{d}\right)$$

Distance between adjacent bright bands is given by $x_{n+1} - x_n$, therefore

$$\boxed{\beta = \frac{\lambda D}{d}}$$

Location of n^{th} dark band is given by

$$x_n = \frac{(2n-1)}{2}\left(\frac{\lambda D}{d}\right)$$

Location of (n + 1) th dark band is given by

$$x_{n+1} = \frac{2(n+1)-1}{2}\left(\frac{\lambda D}{d}\right)$$

Distance between adjacent dark bands is given by $x_{n+1} - x_n$ therefore

$$\boxed{\beta = \frac{\lambda D}{d}}$$

Interference pattern

Alternate bright and dark bands or fringes are observed on screen.

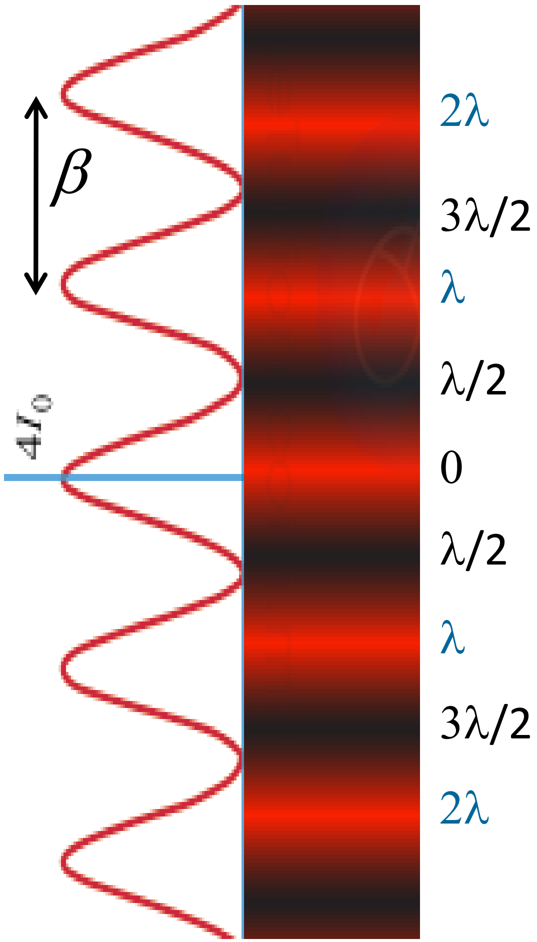
$$\beta = \frac{\lambda D}{d}$$

β increases with increase in D and λ

β decreases with increase in d

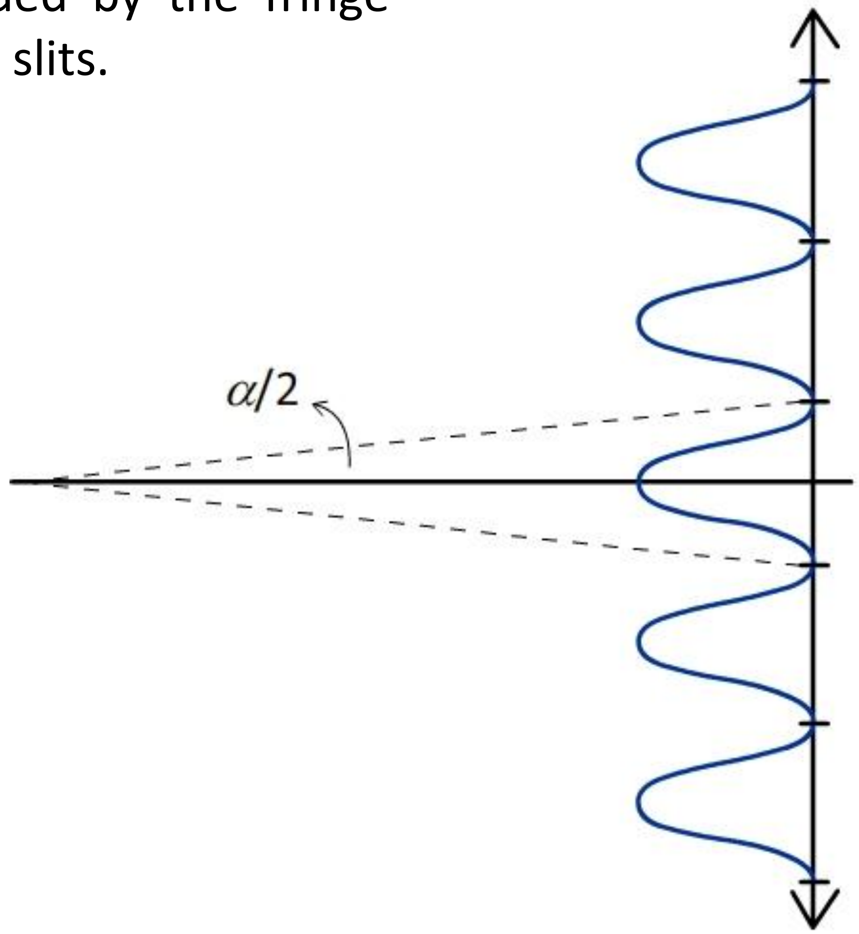
When white light is used, the central fringe (located at 0) is white, followed by higher order coloured fringes

When experiment is performed in a liquid of R.I. μ then β decreases.



Angular fringe width

Angular fringe width is the angle subtended by the fringe width at the midpoint of the line joining the slits.



$$\tan(\alpha/2) = \frac{\beta/2}{D}$$

$$\tan(\alpha/2) = \frac{\lambda D}{2Dd}$$

$$\tan(\alpha/2) = \frac{\lambda}{2d}$$

For small angles $\tan(\theta) \approx \theta$ therefore

$$\frac{\alpha}{2} \approx \frac{\lambda}{2d}$$

$$\boxed{\alpha = \frac{\lambda}{d}}$$

Significance of YDSE

- It established the wave nature of light (thereby supporting Huygens's wave theory)
- Using YDSE, wavelength of light was determined.
- Refractive index of material can be determined using YDSE.
- Thickness of very thin transparent films can be determined using YDSE.

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